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ORIGINAL RESEARCH

Using Digitally Enhanced Tangible Materials for Teaching Fractions: Results of a Project

Emmanuel Fokides[1] · **Konstantinos Alatzas**[1]

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Abstract
Primary school students have trouble understanding concepts related to fractions. On the other hand, technology constantly provides interesting tools that stimulate students' interest and foster learning. Among these, tangible user interfaces allow users to interact with digital applications through the manipulation of everyday objects. Given that conventional tangible materials are already used in the teaching of fractions, the study at hand presents the results of a project in which their impact on learning was compared to that of materials digitally enhanced with the use of tangible user interfaces. The results indicated that the learning outcomes were better for the group of students who used the latter. Moreover, students' enjoyment was greater. However, there were no differences in terms of motivation, ease-of-use, and subjective learning effectiveness. The results can be attributed to both the characteristics of tangible user interfaces and the teaching framework that was followed. Nevertheless, the educational potential of tangible user interfaces has to be further explored.

Keywords Makey-Makey · Mathematics · Tangible materials · Primary school students

1 Introduction

Mathematics is an important teaching/learning subject. They contribute to the development of logical thinking, to the understanding of the natural world, as well as to the organization of our lives. However, students face serious difficulties in understanding and managing mathematical concepts. As a result, they develop negative feelings and anxiety for this subject; they perceive it as a pointless sequence of abstract processes and methods that they simply have to memorize and follow (Holm et al., 2017). On the other hand, students' interest increases when they realize that Mathematics have practical applications and that they can provide solutions to everyday problems (Mazana et al., 2019). To achieve this, the

✉ Emmanuel Fokides
fokides@aegean.gr

Konstantinos Alatzas
premnt19001@aegean.gr

[1] Department of Primary Education, University of the Aegean, Rhodes, Greece

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teaching of Mathematics should be based on students' prior knowledge, mental abilities, and personal interests (Mazana et al., 2019).

Among the mathematical concepts that students, especially the younger ones, find difficult to comprehend, are fractions (Pedersen & Bjerre, 2021). While natural numbers can be easily understood through experience, this does not hold true for fractions (Siegler et al., 2013). That is because students need to realize the relationship between numbers and quantities, the relationship between numerators and denominators, but also the various functions and interpretations that fractions have (George, 2017). To help students, a variety of teaching aids are used, tangible materials being among them. The Grounded Cognition theory, the Embodied Cognition theory, as well as the Concrete-Representational-Abstract (CRA) model provide the theoretical basis for their use. In short, these theories postulated that when learners are tangibly engaged with objects, their understanding of concepts related to these objects is improved (Lindgren et al., 2016).

The role of technology in education is well-established. Out of the various technological artifacts having an interesting potential in the teaching of Mathematics, tangible user interfaces (TUIs) draw our attention. TUIs are devices that allow real objects and digital data to be connected; users can interact with the digital world by manipulating real objects (Nathoo et al., 2020). While the usefulness of tangible materials (henceforth, conventional tangible materials, CTMs), as well as the importance of embodiment in the context of learning, has already been adequately studied, with Mathematics not being an exception (e.g., Furner & Worrell, 2017; Morrissey & Hallett, 2018), this does not hold true for TUIs. A very simple and easy-to-use TUI, used in education with noteworthy results, is Makey-Makey (e.g., Abrahams, 2018; Fokides & Papoutsi, 2020; Hijón-Neira et al., 2020; Palaigeorgiou et al., 2017).

Considering the above and taking into account that the research involving the use of TUIs in the teaching of subjects related to fractions is rather limited, we decided to implement a project, having as an objective to examine whether primary students' performance when learning about fractions with the use of tangible materials enhanced with Makey-Makey (henceforth, digitally enhanced tangible materials, DETMs) is better, compared to the use of CTMs. In addition, we decided to examine the views and feelings of students regarding the use of DETMs. In the following sections, we present the existing research on fractions and TUIs, the reasoning behind the research hypotheses we formulated, the project's setup, the results from its implementation, and their subsequent discussion.

2 Background

2.1 The Teaching of Fractions

Fractions (rational numbers) represent five distinct functions and have an equal number of interpretations (Beyranevand, 2014):

- Part of the whole. In this form, fractions are usually illustrated as objects that are broken into pieces of equal size. Therefore, fractions represent the comparison between the number of the selected pieces (numerator) and the total number of the object's pieces (denominator). Students need to understand that (1) all the pieces when put together produce the whole, (2) the more the pieces the whole is divided to, the smaller these pieces are, and (3) the relationship between the parts and the whole remains the same, regardless of the shape, size, layout, or orientation of the individual pieces. Although the notion of fractions as part of the whole is fundamental to the teaching of fractions, its extensive use creates confusion among students, as it removes them from the mathematical nature of fractions and their algebraic logic (Middleton et al., 2015).

- Ratios. Fractions as ratios express the relationship between two quantities, their relative magnitude. This is useful for size comparisons. In order for students to understand this interpretation of fractions, they should be able to realize the correlation between the two quantities and how these quantities are able to change simultaneously.
- Operators. Fractions as operators or multipliers apply to numbers, geometric shapes, and collections of distinct objects, transforming them by a certain size. With this interpretation, students can better understand the equivalence and multiplication of fractions.
- Indicated quotients. In this case, fractions are the result of dividing their numerators by their denominators. What is important, is that different subsets can be used in the numerator and denominator. For example, the numerator can express objects and the denominator people to share the objects with. In order to understand this interpretation of fractions, activities with fair-sharing problems are used.
- Measures. This expression of fractions is considered the most difficult one as, in this case, fractions act as numbers and not as a relationship between numbers. When placed in a number-line, fractions can be used as reference points for measuring different distances on it. Thus, the basic properties of rational numbers can be represented, such as their density, uniqueness, sequentially, and infinite number.

The above functions are not adequately elaborated in all levels of education (Vlachou & Avgerinos, 2019). In fact, fractions are a source of difficulties and confusion for students, while researchers have been investigating this issue for years. There are several reasons behind students' difficulties. For example, Moseley and Okamoto (2010) reported that some students do not grasp the multiple expressions of fractions and focus on their superficial similarities, rather than their numerical meaning. Students' prior understanding of natural numbers and the complexity of fractions play a significant role. Indeed, Christou and Vamvakoussi (2021) supported that students rely on their knowledge on natural numbers to interpret information about fractions. They also supported that the size of the numbers involved in the operations affected students' evaluations for fractions. Regarding the ranking of fractions, students tend to believe that if the numbers at the numerator and denominator are big, then the fraction is bigger compared to others (Önal & Yorulmaz, 2017). The "density" of fractions also poses some problems, as students cannot understand this notion; contrary to natural numbers, between two fractions there is an infinite number of fractions and a fraction can be equal to an infinite number of other fractions (Markovits & Sowder, 1994).

Students find the rules for the operations between fractions more complex than those for natural numbers. For example, instead of understanding fractions, students try to memorize formulae and algorithms (Deringöl, 2019). They also add or subtract the numerators and the denominators at the same time (Stafylidou & Vosniadou, 2004). Students are also confused when multiplying or dividing fractions (Vamvakoussi et al., 2012). This is because when multiplying two natural numbers, the product is always a larger number (and in the division, the quotient is smaller than the dividend). In fractions, the multiplication may result in a smaller number and the division may result in a larger one. Finally, errors and misunderstandings due to incorrect teaching are not uncommon; teachers should take into account students' experiences and their conceptual understanding and avoid the "quick" teaching of abstract symbols, as these may lead to misconceptions (Aliustaoğlu et al., 2018).

A logical question that emerges is how to effectively teach fractions. Usually, the teaching starts with the expression of fractions as part of the whole, as this constitutes the basis for understanding the other functions (Ramadianti et al., 2019). In fact, Lamon (2007) argued that the simultaneous teaching of all fraction functions should be avoided, as this will lead to superficial knowledge. For easing students' misconceptions, Deringöl (2019) proposed to include pieces of identical problems and the teaching to be student-centered. Another way to achieve an adequate conceptual understanding of fractions is through multiple representations. That is because multiple representations contribute to the conceptual understanding of fractions and fraction operations (Kara & Incikabı, 2018). There are five categories of representational systems that can be used in the teaching and learning of fractions (Behr et al., 1983): (1) real world and everyday life situations, (2) tangible materials, (3) images and diagrams, (4) the spoken language, and (5) the symbols used for writing fractions. Then again, using a single representation system and focusing on a single function of fractions, can have a negative impact. For example, conventional teaching focuses on fractions as part of the whole, making this function the most mastered one by students (Kolar et al., 2018), leaving the rest not so well taught. The number line is also a representational system, in which fractions are expressed as measures. It is a very easy-to-use tool, which not only combines algebra with geometry, but can be used for interpreting integers, rational, and negative numbers. Because the number line is divided into equal parts, visualization is easier, compared to other representational systems such as pies (Wu, 2011). For instance, it is easier to compare fractions by placing them on the number line. Unlike fractions can also be compared by using different number-lines. As a result, the number-line helps students to understand the quantities represented by fractions (Tian & Siegel, 2017), the relationship between fractions and between fractions and integer numbers, the way integers are written as fractions, and how improper fractions are created (Palaigeorgiou et al., 2018).

2.2 (Conventional) Tangible Materials in Mathematics Education

As we mentioned in the preceding section, tangible materials are a representation system that can be used in the teaching of fractions. In primary education their use is frequent, offering enjoyable experiences (Moyer, 2001), increasing students' creativity, and allowing teachers not to rely exclusively on worksheets (Furner & Worrell, 2017). They can be ready-made ones, or created by teachers and students. For example, they can be fake coins, buttons, Lego-like bricks, Cuisenaire rods, tangrams, and geoboards. We have to note that the sole use of tangible materials does not guarantee positive results. Factors such as the teacher's beliefs and knowledge of their use, as well as the educational context, are very important (Moyer, 2001).

According to Piaget (2013), the use of tangible materials is encouraged when teaching Mathematics, because children do not have a well-developed ability to perceive abstract mathematical concepts presented with letters and symbols. By using tangible materials, students gain experiences that allow them to see the connections that exist between materials and concepts. They also provide opportunities for communication between students and teachers. The above, ultimately lead to the construction of knowledge, as well as to the progression of students' existing mathematical ideas to a higher level (Kontas, 2016).

The CRA model, that is based on Bruner's and Kenney's (1966) representation stages, proposed four basic principles regarding the use of tangible materials in the teaching of Mathematics (Furner & Worrell, 2017):

- Their use should not be sporadic; they should be constantly used so as students to familiarize themselves with this type of learning.
- They should be used at the early stages of the teaching process; the abstract concepts should follow and be introduced gradually.
- Materials that are very similar to everyday objects should be avoided. Also, the materials should not have characteristics that can distract students.
- The relationship between the materials and the mathematical concepts should be explained in an explicit and effective way.

The Grounded Cognition theory and the Embodied Cognition theory also provide theoretical support for the use of tangible materials. In short, the former postulated that cognition is triggered by bodily states, modal simulations, and situated action (Barsalou, 2008). For example, numbers can be represented by using fingers and hands. In fact, finger counting affects numerical cognition (Morrissey & Hallett, 2018). The Embodied Cognition theory extended this idea by suggesting that physicalizing processes is the conceptual foundation for knowledge building (Lindgren et al., 2016). The tangible engagement with objects immerses learners into meaningful activities and increases their engagement, leading to the understanding of abstract concepts (Eguchi, 2016); this holds true even for "difficult" learning domains such as Mathematics and sciences (Fokides & Papoutsi, 2020). Others shifted the focus to the manipulability of objects, arguing that overly rich physical activities are a distraction factor (Zacharia & Olympiou, 2011). Nevertheless, there is a consensus that practice through physical manipulatives positively impacts problem-solving skills, collaboration (Johnson et al., 2016), and knowledge retention (Carbonneau et al., 2013).

2.3 Tangible User Interfaces, Makey-Makey

TUIs can be used in all educational settings and there is an increasing interest in their impact on learning (Fuccio & Mastrobeti, 2018). The same theories justifying the educational use of CTMs, apply to TUIs as well. Their purpose is to enhance the interaction between humans and computers by taking into account the multimodality with which humans interact with their environment. They achieve this by connecting real objects to digital data and, thus, augmenting reality. TUIs simultaneously function as input and output devices, allowing users to tangibly, visually, and audibly recognize that a process has been started or completed.

According to Zhou & Wang (2015), the factors that render TUIs effective educational tools are physical, social, and emotional. For example, they are easy to use (Hershman et al., 2018; Rogers et al., 2014), they foster collaboration, and they allow for a pleasant learning environment (Zaman et al., 2012). Moreover, they offer increased opportunities for learning and exploration, allowing students to become active learners (Almukadi & Boy, 2016). In addition, they foster social interactions as well as physical activities (Yu et al., 2020). In addition, Antle (2013) found that the use of TUIs offered an enjoyable experience, improved students' communication skills, and reduced their cognitive load, more than digital or board games. High levels of enjoyment and cooperation were found in Sapounidis et al.'s (2019) study. The authors concluded that these, together with the lack of usability issues, contributed to the positive learning outcomes. Morita and Setozaki (2017) concluded that TUIs allowed primary school students to discover knowledge by themselves and better understand complex concepts (i.e., the Moon phases) compared to videos and a teacher-centered teaching method. TUIs also had high acceptance rates (Fuccio & Mastrobeti, 2018), helped students to develop their metacognitive skills, as well as a positive attitude towards learning (Fleck et al., 2018). Lin et al. (2020) noted a positive impact on computational thinking, motivation, and classroom behavior. Then again, there are some considerations that need to be taken into account. Zhou & Wang (2015) highlighted the difficulties in programming them and the fact that they are not so flexible as other digital tools, rendering the use of the same TUIs (and the applications developed for them) to different teaching subjects a challenging task. They also stressed the lack of a pedagogical framework for their use and the need for students to adapt to a teaching/learning style quite different from the one that they are already familiar with.

There is also research in the context of Mathematics education, although limited. For example, Zito et al. (2021) tested two TUIs and concluded that they can be used in various scenarios and by students of a wide age range. Pires et al. (2019) focused on problem-solving tasks. They found that TUIs helped students to become more active, which lead to a greater understanding of mathematical concepts. Similarly, Chaliampalias et al. (2016), who examined young students' problem-solving skills (related to addition), concluded that TUIs assisted the conceptual transfer.

A TUI specifically developed for education is Makey-Makey (https://makeymakey.com/). It is a very simple, small, and cheap circuit board, connected to a computer through a USB port (see Fig. 2, section "Materials and apparatus"). It is automatically recognized as a human–computer interface; there is no need to install drivers or software. With the use of cables and alligator clips, it can be connected to any conductive material. Because it uses highly sensitive resistance switching, it can sense closed circuits even if it is connected to low conductivity materials (e.g., dried fruits and skin). When a switch closes, Makey-Makey interprets that as a mouse click or as a pressing of a specific key. In turn, this can be used as an input command by any software.

In the context of primary education, Makey-Makey, by itself or together with other devices/applications, has been used for the teaching of Physics (Fokides & Papoutsi, 2020), History and Geography (Xefteris & Palaigeorgiou, 2019), music (Julia et al., 2019), programming (Hijón-Neira et al., 2020; Pérez-Marín et al., 2019), for language learning (Choosri et al., 2017), for enhancing creativity (Abrahams, 2018), and for learning the time (Palaigeorgiou et al., 2017). It has also been used in the context of STEM education (Chen & Lo, 2019; Hsu et al., 2018; Marín-Marín et al., 2020), in students with special needs (Aydogan & Aydogan, 2020; Lin & Chang, 2014), but also for enhancing the mental abilities of elderly people (Rogers et al., 2014). As with other TUIS, most of the above studies reported a positive impact on learning, skill acquisition, engagement with the learning subject, motivation, and enjoyment.

On the other hand, although the use of Makey-Makey for teaching Mathematics (and especially fractions) to primary school students seems logical, we were able to locate an extremely limited number of studies in this field (e.g., Mpiladeri et al., 2016); the rest of the literature either proposed prototypes (e.g., Molina-Villarroel et al., 2021) or examined how pre-service teachers can use Makey-Makey (e.g., Matthews et al., 2018).

2.4 Statement of the Problem and Hypotheses Formation

Reflecting on the research we presented in the preceding sections, we can conclude that TUIs can be used in a wide range of teaching scenarios and that they have an interesting educational potential. However, it is also true that research on this field is still rather unsystematic. Moreover, their use in Mathematics education is limited. Not only that, but most of the relevant research seems to be limited in testing initial ideas, the sample sizes were small, the number of interventions was also limited, and comparisons with alternative tools were not that common. In addition, we were not able to locate many studies in which participants' prior knowledge was taken into consideration.

Having these in mind, we decided to implement a project to answer whether DETMs have a measurable impact on primary school students' learning regarding fractions and whether the results are better (or worse) compared to CTMs, which are commonly used in the teaching of this subject. Moreover, we considered it important to examine students' feelings and opinions regarding DETMs, namely if they consider their experience when using them as being motivating and joyful, whether they find them easy to use, and whether they think that they are useful learning tools. Thus, we formulated the following research hypotheses:

H1 When teaching subjects related to fractions to primary school students and after controlling for their initial knowledge level, digitally enhanced tangible materials, produce better learning outcomes compared to conventional tangible materials.

H2a–d Compared to conventional tangible materials, primary school students think that digitally enhanced tangible materials: (a) offer a more enjoyable experience, (b) are more useful in their learning, (c) are easier to use, and (d) are more motivating.

3 Method

Given that we wanted to compare the results from the use of CTMs and DETMs, we decided to follow a between-subjects research design with two groups. This means that the same subjects were taught to both the control group (CTMs) and the experimental group (DETMs). We have to stress that our project was caught in the midst of the turmoil caused by the COVID pandemic. This resulted in numerous restrictions and uncertainties, related to how many students we could enroll and the number of interventions/sessions we could implement. We discuss these limitations together with the other details of our project in the following sections.

3.1 Participants and Duration

One of the considerations we had was related to the required sample size. Because of the pandemic, many parents decided to keep their children at home or infrequently sent them to school. Thus, it was inevitable to have problems recruiting many students. Nevertheless, our objective was the number of participants to allow us to detect large effect sizes but with more than enough power. For that matter, we performed a power analysis for sample size estimation using G*power (Faul et al., 2007). Following Cohen's (2013) guidelines, for $f_{Cohen} = 0.40$, $\alpha = 0.05$, and $power = 0.90$, the projected sample size was at least sixty-eight participants.

Another decision we had to make was related to participants' age. According to Greece's program of study for primary schools, subjects related to fractions are taught in grades three through six, while in the latter grade they are taught in a systematic way, putting emphasis on problem-solving. Thus, we decided sixth-grade students to be our target group (ages eleven to twelve). We contacted several sixth-grade teachers working in public primary schools in Veria (a middle-sized city in northern Greece, not so hardly hit by the pandemic) and asked them if they were willing to participate in our project. As a result, we selected four classes with a total of eighty students (with the number of girls being slightly more than that of boys) who: (1) were never formally taught the subjects discussed in our study (2) had never before used DETMs, and (3) in terms of their academic performance (as assessed by their grades in previous classes) they were equally divided into three categories (i.e., low, average, and high performance), and with -more or less- an equal number of boys and girls in each group. Half of the students were assigned to the control group and the other half to the experimental.

Because the project involved minors, we applied and we were granted an ethical clearance from the University's ethical committee. In addition, we informed students' parents of the project's objectives and they provided us their written consent for their children's participation.

The project lasted for twelve two-teaching-hour sessions (six for each group), from late October 2020 to late December 2020.

3.2 Materials and Apparatus

Sixth-grade's Mathematics textbook was the starting point for developing our teaching material. Out of the chapters it includes related to fractions, we selected six, namely the comparison of like and unlike fractions, the addition and subtraction of like and unlike fractions, and comparison of proper and improper fractions.

For both the CTMs and DETMs groups the main materials we used were cardboard pieces of various colors that, when put together, formed shapes (e.g., square, oblong, circle, triangle, hexagon, and octagon). The only difference between the cardboard pieces of the CTMs and DETMs group was that the bottom side of the latter was covered with aluminum foil, for conducting electricity (Fig. 1). For the number-line, we used (1) paper strips of various sizes and colors (CTMs group), (2) strips of Play-Doh

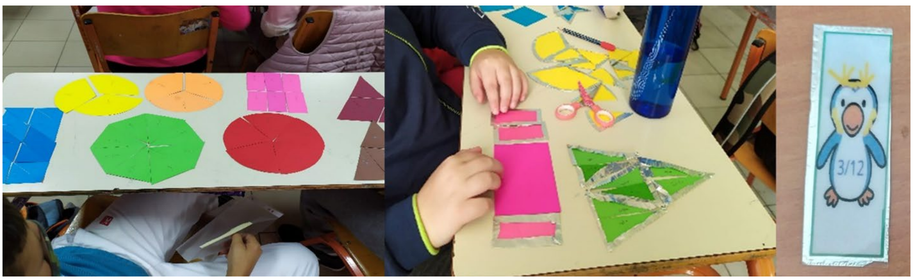


Fig. 1 Cardboard pieces and cards used in CTMs and DETMs groups

plasticine which can conduct electricity (DETMs group), and (3) cards (covered or not covered with aluminum foil, for both CTMs and DETMs groups). We have to note that in the chapters/sessions in which unlike fractions were discussed, we included additional cardboard pieces, so as students to be able to convert the unlike fractions to their equivalent proper ones.

For the CTMs group, we wrote worksheets that included exercises and activities to be conducted during teaching (as explained in the "Procedure" section). As for the DETMs group, the exercises and activities were converted into mini-applications using Scratch. We have to note that when writing the worksheets and during the development of the mini-applications, we took into consideration the difficulties students have in fractions (as we presented them in the section "The teaching of fractions"). For determining what exercises/activities to include, the participating teachers together with the researchers contributed to an initial pool of exercises. These were later discussed in consecutive meetings, in which their purpose, significance, and difficulty level were assessed. The objective was to maintain a balance between easy and difficult exercises and to cover all aspects of the subjects students were taught. The links for the mini-applications can be found in Appendix 1.

We also devised two versions of a device we called "FractionPad." Its dimensions were around 40 X 30 cm and it was made out of cardboard, pins, and double nails (Fig. 2). At the bottom, we placed the number-line. At the top right corner, there was the "OK" button, that students could touch after finishing an activity or exercise, so as to check the result. At the top left corner, we placed four arrows that could be used as a joystick, for example, for selecting an answer to a question (version 1) or several areas in which students could place cardboard pieces for making fractions (version 2). Using cables, all the above were connected to a Makey-Makey, which, in turn, was connected to a computer. Each pair of students had a computer and a FractionPad at their disposal. Following the instructions displayed in the mini-applications, students used their FractionPads to conduct the exercises and see the results in the mini-applications (Fig. 3). They could also keep notes for the discussions that followed (see "Procedure" section).

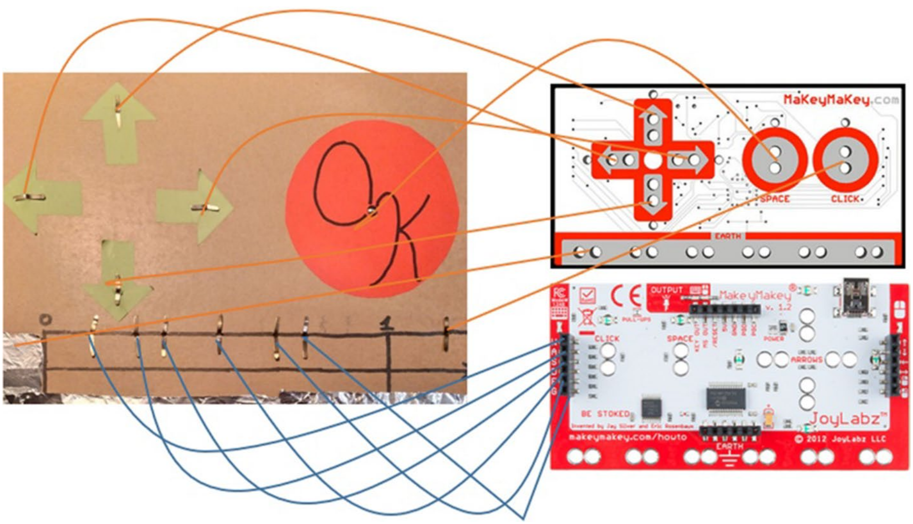


Fig. 2 The FractionPad

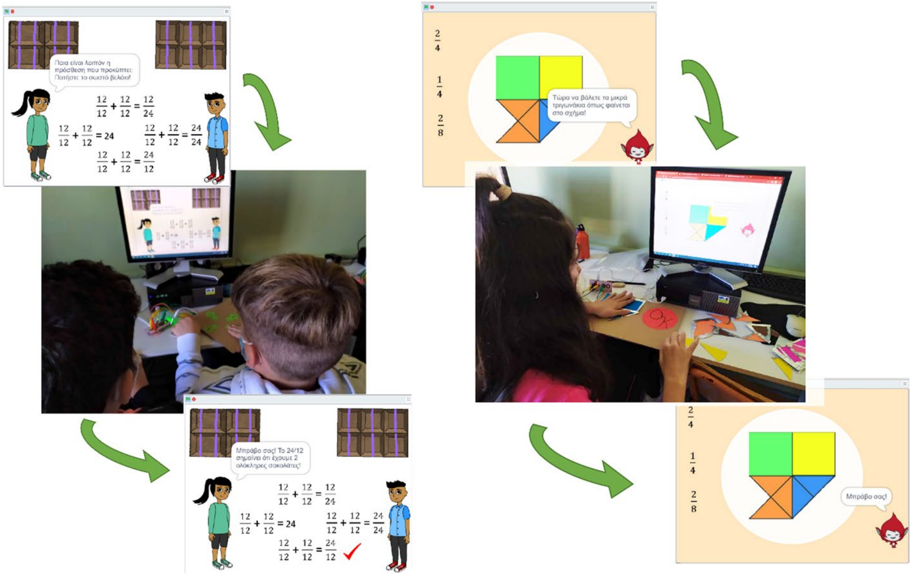


Fig. 3 The FractionPad and the mini-applications in action

3.3 Procedure

Freudenthal (2012), the founder of Realistic Mathematics Education (RME), argued that it is preferable for students to be taught Mathematics in such a way that it is useful in their daily lives. In this respect, the starting point of teaching should be authentic situations (real or imaginary) that make sense to students, rendering Mathematics an activity rather than a simple learning subject.

Six principles are at the core of RME teaching (Van den Heuvel-Panhuizen & Drijvers, 2020):

- Activity principle. This principle emphasizes the concept of active student participation, as well as the practical aspect of mathematical activities.
- Reality principle. The real-life and the theories of Mathematics have to be connected. Teaching takes place with students exposed to problematic situations that need to be solved and not through theorems.
- Level principle. In order for students to master knowledge, they must go through various stages of understanding.
- Intertwinement principle. Mathematics are considered and treated as a single entity and not as individual /unconnected concepts. The various subjects of Mathematics overlap.
- Interactivity principle. The teaching of Mathematics is not an individual activity, but a social one.
- Guidance principle. This principle reflects teachers' proactive role; their long-term agenda should be the change of students' beliefs about Mathematics.

Problem Based Learning (PBL) supports similar ideas. The problem-solving approach is carried out in an integrated activity framework aimed at developing mathematical concepts. Cooperation and group-work in an environment that is open and safe for all, are

PBL's key features (Schettino, 2016). Communicating beliefs, values, knowledge and skills, counseling, and research, are also important (Tan, 2003). The above motivate students and help them to develop their higher mathematical thinking, as well as the ability to connect and apply the relevant concepts to real-life situations. PBL suggests that (Tan, 2003):

- The starting point of teaching is a problem that has to be authentic and as close to real-life as possible, so as to stimulate the interest of students who then try to solve it.
- Viewing the subject from multiple perspectives is important. Interdisciplinarity should be encouraged and knowledge from different learning domains should be included.
- The problem has to challenge students' existing knowledge/perceptions and help to expand them into new areas.
- Students have to be allowed to take command of their own learning. They should be responsible for the information they obtain and process.
- Learning is a collaborative process, based on communication. Students should work in small groups.
- When the problem under investigation is solved, summarizing and consolidating what students learned is required. Students have to be involved in activities that try to merge the new knowledge with the existing one.
- In order for teaching to complete, an evaluation and review of what was done must be carried out.

According to the CRA model (see section "Conventional tangible materials in Mathematics education"), teaching must begin with tangible materials for the initial understanding of a concept. In the second stage, it is proposed to use different representations of the concepts, such as images and diagrams, while, in the third stage, the teaching involves abstract concepts and symbols. The ultimate goal is for students not to have the need to use tangible materials any longer (Flores et al., 2014).

Active students' participation in the learning process, learning based on solving problems related to real-life situations, multiple representations of a concept, cooperation/discussions/communication of ideas, gradual introduction of more abstract concepts, and the use of tangible materials during the initial teaching stages, are all teaching ideas proposed by RME, PBL, and the CRA model. Inspired by them, the teaching procedure we followed involved five stages:

- Introduction-discussion of the concepts to be taught (10 min).
- Teaching using CTMs or DETMs and number-line (40–50 min). Students used the materials (either CTMs together with worksheets or DETMs together with the mini-applications) for completing the exercises/activities. They were free to discuss and exchange their ideas. Feedback was provided by the teachers after finishing the exercises (in the CTMs group) or automatically (from the mini-applications in the DETMs group). Following that, the groups presented the solutions to the rest of the class together with their reasoning for reaching the solution. The last exercises involved the use of the number line (either conventional or digitally enhanced) because it is a more abstract system.
- Teaching through problem-solving (15–25 min). This stage did not involve the use of materials. Students were engaged in solving one of two problems derived from everyday life. Once again, the groups were asked to present the solution and their reasoning to the rest of the class.

- Discussion (10 min). For closing the teaching of a subject, students discussed what they have learned, the difficulties they encountered, and how they can implement what they have learned in their lives.

Students worked in pairs. Due to the emergency measures against the pandemic, a larger number of students per group was not allowed. The role of the teachers was supportive throughout the process. Also, they did not provide any direct answers while students were engaged in solving problems or when conducting the exercises/activities.

3.4 Instruments

For collecting data regarding the impact of each tool on students' knowledge (i.e., for examining H1) and since we had six sessions for each tool, we developed an equal number of evaluation tests. For determining what questions to include, we followed the same procedure for the development of the exercises/activities. Moreover, we decided to include, in each test, a -rather complex- problem drawn from students' everyday life. In Appendix 2 we present some examples of the questions included in the evaluation tests. We administered each test immediately after the end of its corresponding session. In addition, for establishing participants' prior knowledge, we tested them using a pre-test, having questions similar to the ones in the evaluation tests, that covered subjects included in all sessions. We administered the pre-test a week before the beginning of the project.

For examining H2a–d, we used four factors included in a modular scale developed for recording users' experiences when using digital educational tools (Fokides et al., 2019), namely enjoyment, subjective usefulness, ease-of-use, and motivation. The items, nineteen in total, were presented on a five-point Likert-type scale (ranging from strongly disagree = 1 to strongly agree = 5). The questionnaire's items can be found in Appendix 3. We administered the questionnaires during each group's last session.

4 Results

As we presented in the preceding section, a total of eighty students (eleven to twelve years old) participated in our study, who were taught subjects related to fractions using two different tools (CTMs and DETMs). We had to exclude eight students because they were absent in more than one session; thus, our final sample consisted of seventy-two students. For analyzing the results in the pre- and evaluation tests, we graded them on a scale ranging from 0 to 100. Given that we administered six tests per tool, we calculated the average students' scores in each tool and we imputed the resulting data in SPSS 26 for further analysis. We present descriptive statistics for the tests' results in Table 1.

Because we wanted to control for the effects of students' prior knowledge, we considered an analysis of covariance (ANCOVA) as being the appropriate method for analyzing our data. Prior to doing so, we checked whether they met the assumptions for this type of statistical analysis and we noted no problems. We examined the results using an alpha of 0.05. As it is evident in Table 2, there is indeed a statistically significant difference between the two tools and the effect size is very large ($F = 33.14$, $p < 0.001$, $\eta^2_p = 0.332$). Thus, we can confirm H1; the use of DETMs for teaching primary school students subjects related to fractions produced better learning outcomes compared to the use of CTMs.

Table 1 Descriptive statistics for the pre-test and the evaluation tests

Tool	*n*	*min*	*max*	*M*	*SD*	95% confidence interval	
						Lower bound	Upper bound
Pre-test CTMs	36	13.00	77.00	30.28	13.62	25.67	34.89
Pre-test DETMs	36	12.00	80.00	31.56	14.19	25.51	35.05
Evaluation tests CTMs	36	19.25	89.75	52.06	16.10	46.61	57.50
Evaluation tests DETMs	36	14.00	100.00	73.86	20.57	66.90	80.82

Table 2 The results of the ANCOVA test

Source	*SS*	*df*	*MS*	$R^2{}_{adj}$	*F*	*p*	$\eta_p{}^2$
Pre-test	6058.27	1	6058.27		23.46	< .001	0.254
Evaluation tests	8558.68	1	8558.68	0.435	33.14	< .001	0.324
Residuals	17,817.41	69	258.22				

Table 3 Descriptive statistics for the questionnaires

Factor	CTMs				DETMs			
	M	*SD*	95% conf. interv		*M*	*SD*	95% conf. interv	
			L. bound	Up. bound			L. bound	Up. bound
Enjoyment	3.72	0.87	3.42	4.01	4.31	0.51	4.14	4.48
Subjective usefulness	3.80	0.76	3.54	4.06	4.01	0.60	3.81	4.21
Ease of use	3.91	0.87	3.62	4.21	4.11	0.65	3.89	4.33
Motivation	3.77	0.98	3.43	4.10	4.01	0.83	3.74	4.30

As for the questionnaires, we assessed their overall internal consistency (as well as of their factors) using Cronbach's alpha. As α in all cases was well above the 0.70 threshold (ranging from 0.77 to 0.81), we concluded that their consistency was satisfactory (Taber, 2018). Then, we calculated a total of eight new variables, representing the average of each factor in the two questionnaires (two questionnaires X four factors each). Again, we imputed the resulting data in SPSS 26. We present descriptive statistics for the questionnaires in Table 3.

We considered an analysis of variance (ANOVA) as being the appropriate method for analyzing the questionnaires' data. Once again, we checked whether our data met the assumptions for this type of statistical analysis and we noted no problems. Given that, we proceeded conducting a total of four tests (one for each factor) (Table 4). The only statistically significant difference we noted (and with a large effect size) was in enjoyment ($F = 12.44$, $p = 0.001$, $d_{Cohen} = 0.83$). As a result, H2a is the only hypothesis we can confirm; primary school students enjoyed the use of DETMs in their teaching of subjects related to fractions more than the use of CTMs. On the other hand, H2b, H2c, and H2d have to be rejected. That is because, on the basis of the results presented in Table 4,

Table 4 The results of the ANOVA tests

Factor	*df*	*MS*	*F*	*p*	d_{Cohen}
Enjoyment	1	6.31	12.44	0.001	0.83
Subjective usefulness	1	0.78	1.67	0.201	0.31
Ease of use	1	0.71	1.20	0.276	0.26
Motivation	1	1.13	1.36	0.247	0.26

students considered DETMs and CTMs as being equally motivating and equally easy to use. They also considered that both were equally useful in their learning.

4.1 Additional Analysis

We decided to conduct an additional analysis, in order to examine the impact of the above factors on the learning outcomes when using CTMs and DETMs. For that matter, we run two multiple linear regression analyses using the Enter method. The dependent variable was students' mean scores in the evaluation tests, while the independent variables were the mean scores of the four questionnaires' factors. We advise caution when interpreting the results of this analysis, as our sample was below the threshold the relevant literature recommends. Acknowledging this limitation, the results, as we present them in Tables 5 and 6, demonstrated that ease-of-use had a significant positive impact on students' learning when using CTMs ($t = 2.56$, $p = 0.016$) and DETMs ($t = 2.27$, $p = 0.005$). Enjoyment had a borderline statistical significance in the results of the DETMs group ($t = 1.98$, $p = 0.050$).

Table 5 The results of the regression analysis for the CTMs

Model summary	$F(4, 31) = 2.32$, $p = .079$,,$R = .480$, $R^2 = .230$				
	b	*SE B*	*β*	*t*	*p*
Enjoyment	−5.15	6.97	−0.28	−0.74	0.466
Subjective usefulness	4.29	7.28	0.20	0.59	0.560
Ease of use	10.76	4.20	0.58	2.56	0.016
Motivation	−2.03	5.42	−1.24	−0.38	0.710

b = unstandardized beta coefficients, $SE\ B$ = standard errors for B, β = standardized coefficients

Table 6 The results of the regression analysis for the DETMs

Model summary	$F(4, 31) = 3.91$, $p = .007$, $R = .435$, $R^2 = .189$				
	b	*SE B*	*β*	*t*	*p*
Enjoyment	10.81	5.50	0.38	1.98	0.050
Subjective usefulness	−9.24	5.13	−0.30	−1.80	0.076
Ease of use	11.09	3.86	0.40	2.27	0.005
Motivation	−4.79	4.12	−0.21	−1.16	0.248

5 Discussion

For a tool, digital or otherwise, it is important to validate if and to what extent it affects learning, in order for experts involved in education to be able to make informed decisions for its introduction in education. That being said, our data analysis demonstrated that, indeed, the knowledge gains from the use of DETMs for teaching fractions to primary school students are statistically significantly more than those from the use of CTMs. Furthermore, some thought-provoking findings emerged from the analysis of the questionnaires' data, worthy of further discussion.

The results in the pre-tests are interesting per se. As we mentioned in a previous section, our sample consisted of sixth-grade students (ages eleven to twelve). We also mentioned that Greek primary school students are taught subjects related to fractions for four consecutive grades (three through six). During the fifth grade, all the relevant subjects are introduced, while in the sixth grade the same subjects are repeated with the emphasis being placed on solving problems. The results in the pre-tests indicated that students were able to answer correctly about a third of the questions (see Table 1). This means that students' prior knowledge about fractions was poor. Although we did not examine the underline reasons leading to this situation, this can be interpreted as a problem of the Greek educational system, which focuses on the memorization of rules, methods, and processes. Inevitably, this leads to the poor conceptual understanding of fractions (Vlachou & Avgerinos, 2019) and to disdain for Mathematics in general (Holm et al., 2017). This finding is also an indicator of the multitude of problems students face with fractions as noted by many others (e.g., Moseley & Okamoto, 2010; Ramadianti et al., 2019; Vamvakoussi et al., 2012; Vlachou & Avgerinos, 2019). The results in the DETMs' evaluation tests delineated a very different picture, as they indicated a 134% improvement. The corresponding improvement in the CTMs' evaluation tests was 72%. So, both tools had a noteworthy impact on students' knowledge, but the impact of DETMs was, by far, more conspicuous. Although we are not entitled to generalize this finding to other subjects, it provides a rather good idea about the educational value of DETMs. It is also consistent with the findings of past research, both in the context of Mathematics education (e.g., Chaliampalias et al., 2016; Matthews et al., 2018; Zito et al., 2021) and in the context of other learning domains (e.g., Fokides & Papoutsi, 2020; Hijón-Neira et al., 2020; Xefteris & Palaigeorgiou, 2019).

Thus, what we have to do is to provide plausible explanations for the results in students' learning outcomes. This task is not an easy one, for reasons elaborated below. We had to devise a teaching procedure, because of the lack of a relevant pedagogical framework (Zhou & Wang, 2015). To our view, it was well-grounded in the theories supporting the use of tangible materials (see section "Procedure"). Given that we applied it to both groups, we can assume that its impact was similar. That is because students' active participation, learning based on solving problems related to real-life, multiple representations of a concept, cooperation, and communication of ideas, all core features of the teaching procedure we followed, were independent of the tool students used.

However, there was a difference in the implementation of the teaching procedure, related to how feedback was provided. Although small, it might have played a decisive role. In general, the role of feedback is pivotal in goal achievement, especially when students work by themselves (Hattie & Timperley, 2007; Shute, 2008). In the CTMs group, it was the teachers who provided feedback; students had to wait for the teachers to come to their group or they had to wait until the next teaching stage in which the results were discussed in the whole classroom. In the DETMs group, students received feedback immediately through the mini-applications. Given that guessing was avoided because students had to provide their reasoning for an answer to a problem or exercise, we believe that the immediate feedback allowed them to be more autonomous, as they had more time to reflect on the task they had, to experiment, and try different approaches to solve it.

Past research indicated that TUIs motivate students to learn (e.g., Fleck et al., 2018), which, in turn, positively impacts the learning outcomes (e.g., Fokides & Papoutsi, 2020; Lin et al., 2020). Although the results in the questionnaires provided further evidence that TUIs offered high levels of motivation ($M = 4.01$, $SD = 0.83$), at the same time we did not find any difference between the DETMs and CTMs, meaning that, in our case, TUIs were not more motivating than CTMs. Moreover, in our additional analysis, we did not find an impact of motivation on the learning outcomes of both groups. Thus, motivation has to be ruled out as a valid explanation for the difference in the learning outcomes. Nevertheless, what is encouraging is that both tools motivated students to learn about fractions, given their overall negative attitude towards Mathematics and the significant problems they face in fractions.

Learner satisfaction is considered a significant determinant of a tool's effectiveness. Even though the relevant literature suggested that it is a multifaced construct affected by a variety of factors, depending on the settings, context, and tools (e.g., Stepan et al., 2017; Sun et al., 2008), three factors were routinely examined: (1) ease of use of the given tool, (2) how useful students consider it in relation to their learning, and (3) enjoyment when using it. Research has indicated that students find TUIs easy to use (e.g., Hershman et al., 2018; Rogers et al., 2014). Once again, our results can confirm this ($M = 4.11$, $SD = 0.65$), but, as with motivation, we did not find any significant difference between the DETMs and CTMs. Moreover, we found that ease-of-use had a positive impact on the learning outcomes of both groups. Thus, we can conclude that this factor certainly contributed to the learning outcomes, but both groups equally benefited. On the other hand, we think that two issues might have negatively impacted the views of students regarding the ease of use of DETMs. Firstly, changes in how computers are operated when TUIs are connected are a cause of concern. Students are, more or less, familiar with the regular input devices (i.e., the keyboard and mouse); when asked to work in a completely different way, because of TUIs, they face some problems (Zhou & Wang, 2015). Secondly, students are more familiar with materials that are commonly used in their teaching; this can negatively affect the successful introduction of any ICT device (Biagi & Loi, 2013).

Coming to the subjective usefulness of the DETMs, the results were similar to the ones regarding the ease of use. Even though DETMs were considered highly useful ($M = 4.01$, $SD = 0.60$), as other researchers pointed out (e.g., Fleck et al., 2018; Palaigeorgiou et al., 2017, 2018), we did not find any differences between the two tools. In addition, we found that neither the CTMs nor DETMs had an impact on students' learning. The results in enjoyment are the only ones in which the DETMs had a clear advantage over the CTMs. Enjoyment is considered one of TUIs strongest features (e.g., Abrahams, 2018; Chen & Lo, 2019; Fucci and Mastrobetti, 2018; Fokides & Papoutsi, 2020; Hershman et al., 2018; Palaigeorgiou et al., 2017). Likewise, we noted high levels of enjoyment in the DETMs group ($M = 4.31$, $SD = 0.51$), statistically significantly more than that in the CTMs group. What is of interest is that enjoyment seems to have had an impact on students learning only in the DETMs group (although the significance was borderline) but not in the CTMs group. Therefore, we can confirm, with some reservations, that there is a link between enjoyment and increased knowledge gains when students are engaged in activities in which TUIs are used.

To summarize, in our study we compared two quite similar forms of tangible materials. In this respect, we expected the differences to be subtle, rendering hard the interpretation of the findings. Indeed, the only significant difference we observed was in students' enjoyment. We also theorized that the difference in the way feedback was provided might have also played a role. Then again, the distance between the learning outcomes of the two tools was rather large; the above are not enough for explaining it. Thus, we are inclined to believe that the differences in motivation, ease of use, and subjective usefulness, all in favor of DETMs, while minor, had a notable summative effect.

5.1 Implications for Research and Practice

Our study adds to the existing body of research that examined the impact of TUIs on Mathematics education as it: (1) contrasted the learning outcomes from the use of DETMs to that of CTMs, (2) explored the views of students regarding DETMs, and (3) quantified, though with limitations, the impact of enjoyment, subjective usefulness, ease-of-use, and motivation on the learning outcomes when using DETMs. Due to the above, we can identify a number of implications for professionals and for those involved in education.

In order to produce our DETMs (namely the FractionPads), we used Makey-Makeys, some cheap materials, and we wrote some -rather simplistic- applications using Scratch. This has a positive and a negative side. On the positive side, any educator can easily and without a substantial cost, replicate and improve our FractionPads. In fact, the cost for acquiring an adequate number of Makey-Makeys is not an issue as (1) cheap clones, at a fraction of the cost of the original Makey-Makeys, are available and (2) they can be used for the development of material for many other subjects/courses; thus, their reusability increases their value-for-money. On the negative side, the FractionPads were far from having the look and feel of commercial/professional products. Although the amateurish nature of the DETMs might have had a negative impact on both the learning and views of students, we did not have any alternative solutions; ready-made products are not available. In addition, it is questionable whether educators are willing to learn to program (although Scratch is not that hard to learn) and then dedicate a considerable amount of time and effort needed for the development of such applications. This calls for professionals to take action; it is almost certain that, compared to us, they can better and more innovatively take advantage of Scratch's features or better utilize any other piece of software that allows the development of applications to be coupled with TUIs.

Considering the results from the use of DETMs, we can support that their integration into everyday teaching is advisable. However, any tool, by itself, does not guarantee good results. Therefore, for any given subject, educators have to take into account the context in which they will use DETMs and whether they offer more advantages compared to CTMs or any other teaching aids. Technological "gadgets," such as TUIs, might distract students from what they are supposed to learn. A well-defined teaching framework, such as the one we implemented, might help to avoid that. Time management is essential. The two-teaching hours we dedicated for each session were enough for students to use DETMs and conduct all the activities. It is true that contemporary teaching methods, especially when they utilize some form of ICTs, require more time than conventional teaching. Consequently, there is the need for education policymakers to make the necessary adjustments to primary school's curriculum and program of study. Moreover, it is advisable to equip schools with TUIs; as we already mentioned, their cost is not significant. Finally, education experts have to take action toward the development of applications that make good use of TUIs' potential, given that teachers do not always have the time and skills to do so.

5.2 Limitations and Future Work

Although our study brought into light interesting results, it has limitations that we have to acknowledge. A larger sample size would have guaranteed the reliability of our findings. The age-range we targeted was narrow; we are not able to offer insights regarding the impact of DETMs on younger or older students. We decided to "teach" most of the subjects included in the textbook that were related to fractions. One might argue that the number of sessions was not enough for doing so, given their complexity. Others can support that it might have been better to focus on just a couple of subjects (e.g., adding and subtracting fractions) and examine them more thoroughly. However, it was impossible to add more sessions given the time restrictions imposed by the schools' timetables and the emergency measures due to the COVID pandemic. Not only that but significant deviations from what the official program of study estimates as being enough for the teaching of fractions, would have had a negative impact on the applicability of our study to real-life teaching conditions.

The above limitations can serve as guidelines for future research; larger sample sizes, wider age-ranges, and more subjects in which DETMs can be applied are definitely needed. Qualitative data may provide a deeper understanding for the impact of DETMs. Longitudinal studies can also be considered, given that the novelty effect resulting from the use of a new educational "gadget" usually wears out after some time. Comparisons between DETMs and other digital tools or applications will also provide valuable data regarding their educational potential. Finally, we think that it would be interesting to examine the educators' views about the use of DETMs in their teaching.

6 Conclusion

Tangible materials are quite helpful in the teaching of fractions to young students. Given that, we assumed that technology, though TUIs, can transform them into more effective tools. For that matter, we implemented a project having as an objective to compare the learning outcomes from the use of CTMs and DETMs Furthermore, we proposed and tested a teaching framework for integrating DETMs into teaching. Bearing in mind the results, as well as the limitations of our study, we feel that our assumption proved to be valid. Not only that, but we provided evidence that DETMs offered a more enjoyable experience to students and were as easy-to-use as CTMs. On the other hand, our data suggested that CTMs and DETMs do not differ in terms of how motivating they are and whether students consider them useful in their learning. In conclusion, our study contributes to the growing body of literature regarding the use of TUIs in education. It might prove useful to researchers for understanding their impact on learning. Educators can also consider using them in their teaching.

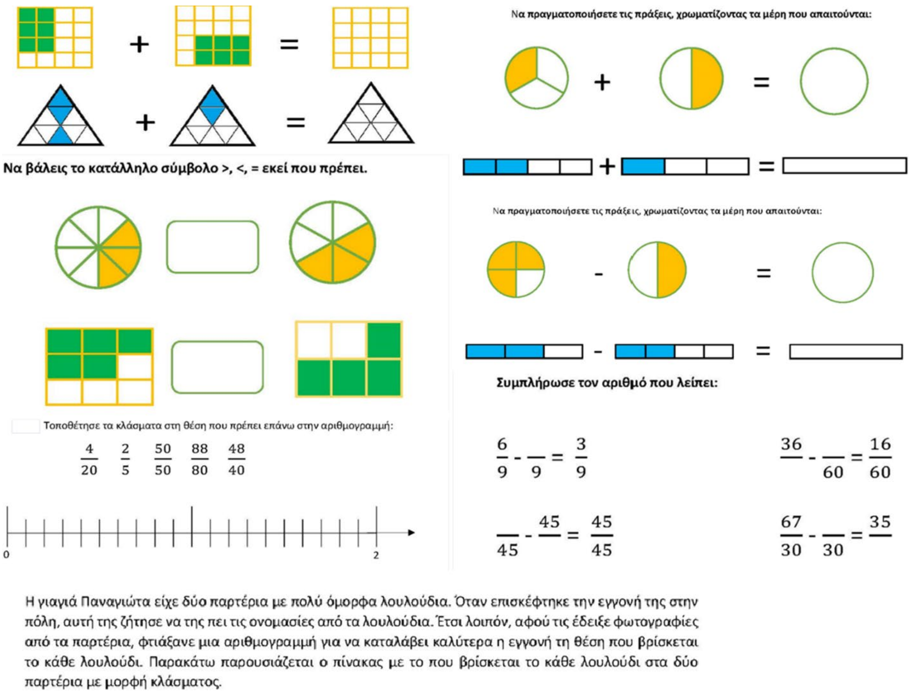
Appendix 1

Links for the Scratch mini-applications developed for the project.

https://scratch.mit.edu/projects/437103568/	https://scratch.mit.edu/projects/444154099/
https://scratch.mit.edu/projects/438278289/	https://scratch.mit.edu/projects/444663859/
https://scratch.mit.edu/projects/439359336/	https://scratch.mit.edu/projects/445707633/
https://scratch.mit.edu/projects/441323099/	https://scratch.mit.edu/projects/449728740/
https://scratch.mit.edu/projects/446890108/	https://scratch.mit.edu/projects/449728740/
https://scratch.mit.edu/projects/452511332/	

Appendix 2

Example questions in the evaluation tests.



Λουλούδια	Θέση που βρίσκονται τα λουλούδια
Γεράνια	$\frac{4}{20}$
Γιασεμιά	$\frac{12}{10}$
Πετούνιες	$\frac{2}{5}$
Γαρύφαλλα	$\frac{6}{4}$
Μολόχες	$\frac{2}{4}$
Τριανταφυλλιές	$\frac{72}{40}$

Βοήθησε την εγγονή της γιαγιάς Παναγιώτας να βάλει τα κλάσματα στη σωστή τους θέση στο κάτω μέρος της αριθμογραμμής, ώστε να κατανοήσει που βρίσκεται το κάθε λουλούδι. Αν θέλεις μπορείς να ζωγραφίζεις και τα λουλούδια επάνω στην αριθμογραμμή, ώστε να την κάνεις πιο όμορφη!

Appendix 3

The questionnaire's items.

Factor	Item
Enjoyment	It was fun to use this tool*
	I felt bored while using this tool**
	I enjoyed using this tool
	I really enjoyed studying with this tool
	I felt frustrated**
Subjective usefulness	I felt that this tool fostered my learning
	This tool was a much easier way to learn compared with the usual teaching
	This tool made my learning more interesting
	I felt that this tool helped me to increase my knowledge
	I felt that I caught the basics of what I was taught with this tool
Ease of use	I think it was easy to learn how to use this tool
	I found this tool unnecessarily complex**
	I think that most people will learn to use this tool very quickly
	I needed to learn a lot of things before I could get going with this tool**
	I felt that I needed help from someone else in order to use this tool because It was not easy for me to understand how to use it**
	It was easy for me to become skillful at using this tool

Factor	Item
Motivation	This tool did not hold my attention**
	When using this tool, I did not have the impulse to learn more about the learning subject**
	The tool did not motivate me to learn**

* = the word "tool" was replaced by "conventional tangible material" and "digital tangible material", depending on the tool students used; ** = item for which its scoring was reversed; all items were presented in a five-point Likert type scale.

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Data availability The data processed in this paper is available when required to authors.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical statement We hereby declare that this manuscript is the result of our independent creation under the reviewers' comments. Except of the quoted contents, this manuscript does not contain any research achievements that have been published or written by other individuals or groups, we are the only authors of the manuscript. The legal responsibility of this statement shall be borne by us.

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